OKLAHOMA STATE UNIVERSITY

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear System
Spring 1999
Final Exam

Name : ______________________________

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To make our future students not to rely too much on the lecture notes, I need your cooperation to agree that you WILL NOT pass on the lecture notes and handouts to students who will take ECEN 5713 in the following semesters. In any event, those who violate this expectation will be punished. The on-line lecture notes in the homepage will be removed immediately after the final exam. Please SIGN below as you concur.

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Problem 1:

Find an equivalent discrete-time Jordan-cononical-form dynamical equation of

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 4 & 3 \\ 0 & 20 & 16 \\ 0 & -25 & -20 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} u(k)$$

.

$$y(k) = \begin{bmatrix} -1 & 3 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + 4u(k)$$

Problem 2:

Let $\lambda_i, i = 1, \cdots, n$ be the eigenvalues of an $n \times n$ matrix *A*. Show that

$$\det A = \prod_{i=1}^{n} \lambda_i \ .$$

<u>Problem 3</u>:

Given is the system of first-order ordinary differential equation $\dot{x} = t^2 Ax$, where $A \in \Re^{n \times n}$ and $t \in \Re$. Determine the state transition matrix $\Phi(t, t_0)$.

<u>Problem 4</u>:

Consider the system representations given by

$$x(k+1) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 0 \end{bmatrix} u(k)$$

and

$$\tilde{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) \, .$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} 0 & 1 \end{bmatrix} u(k)$$

Are these representations equivalent ? Are they zero-input equivalent ?

Problem 5:

Reduce the system with

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 3 & 0 & -3 & 1 \\ -1 & 1 & 4 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

into controller form $A_C = PAP^{-1}$, $B_C = PB$. What is the similarity transformation matrix in this case ?